LOGISTIC REGRESSION

A Powerful Tool for Classification

BASICS OF LOGISTIC REGRESSION

Classification Algorithm:

- **Two categories for the outcome variable (analysed in what follows):** e.g. Unemployed true or false

- Multiple categories for the outcome variable (not covered here)
  - unordered logistic regression
  - ordered logistic regression »

A MOCK-UP EXAMPLE TO INTRODUCE THE IDEA

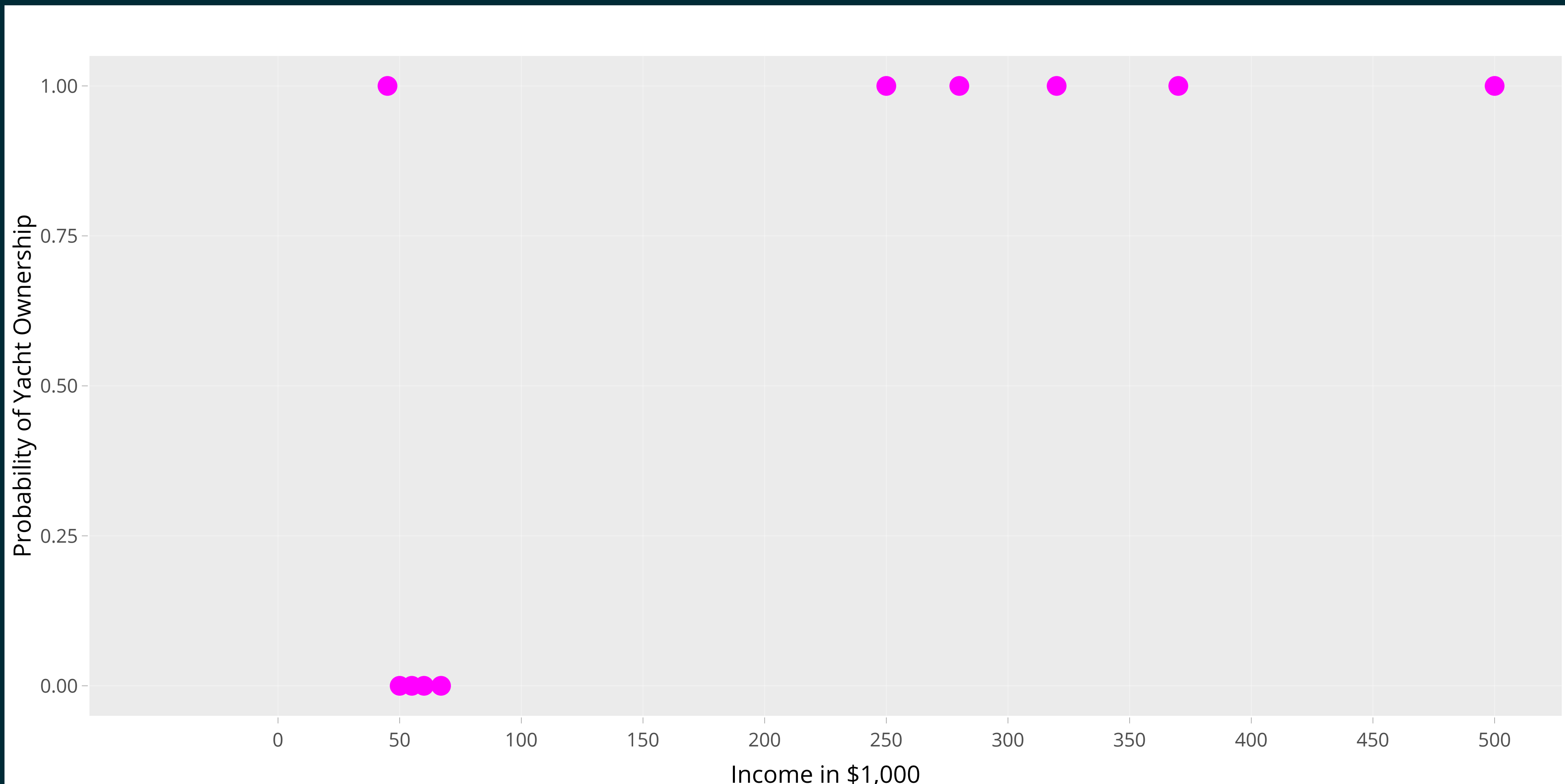
▶ Code

Income and Yacht Ownership

Name	Income	Yacht
Jack	45	1
Sarah	50	0
Carl	55	0
Eric	60	0
Zoe	67	0
James	250	1
Enrico	280	1
Erica	320	1
Stephanie	370	1
Susan	500	1

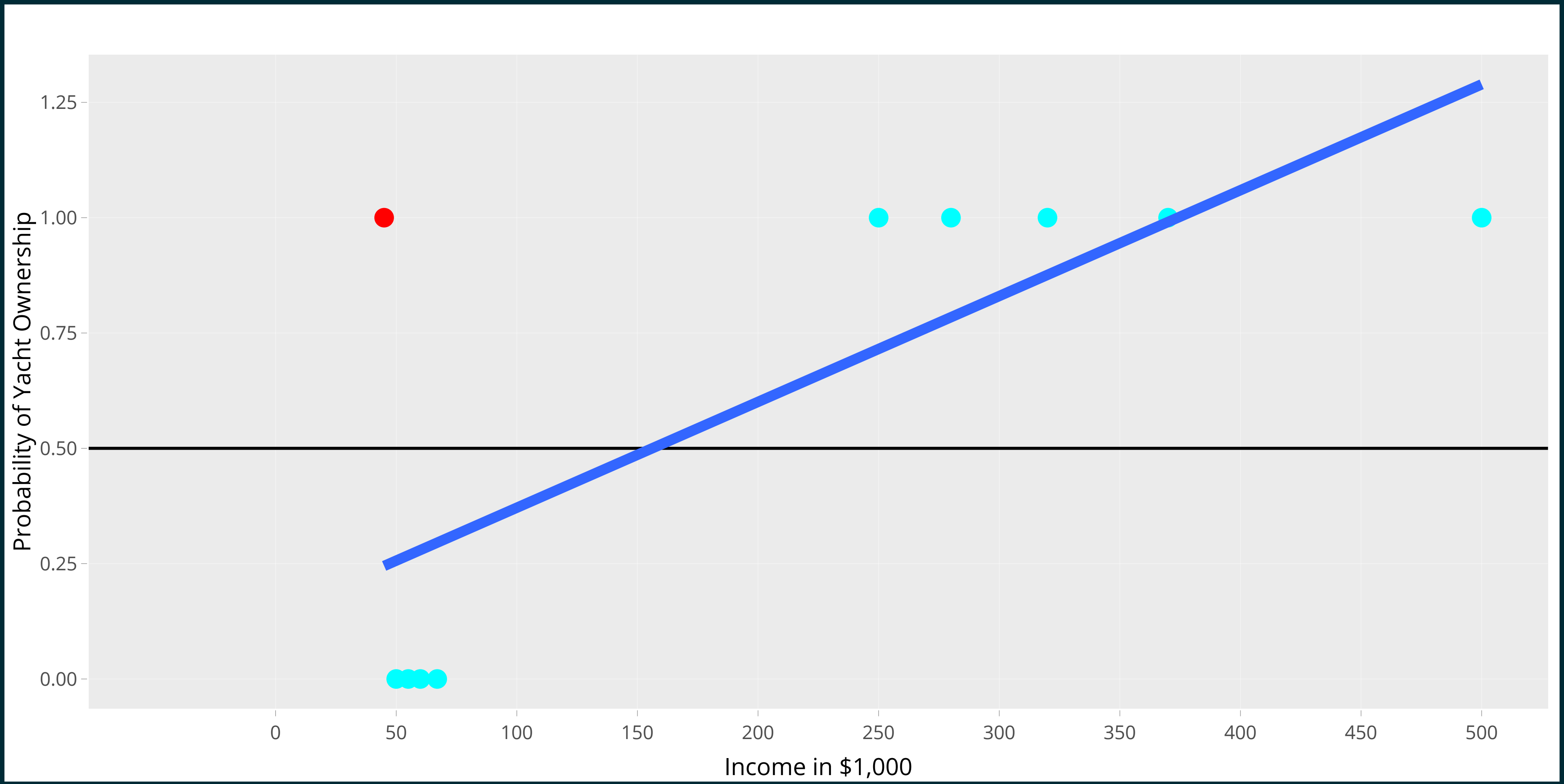
USING OLS IS A TEMPTING (BUT BAD) IDEA

- Code

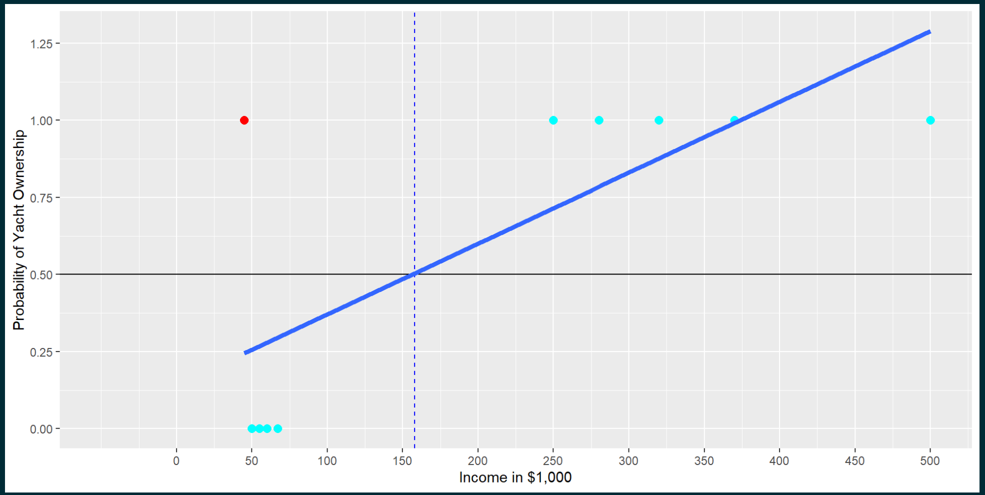


USING OLS IS A TEMPTING (BUT BAD) IDEA

- Code

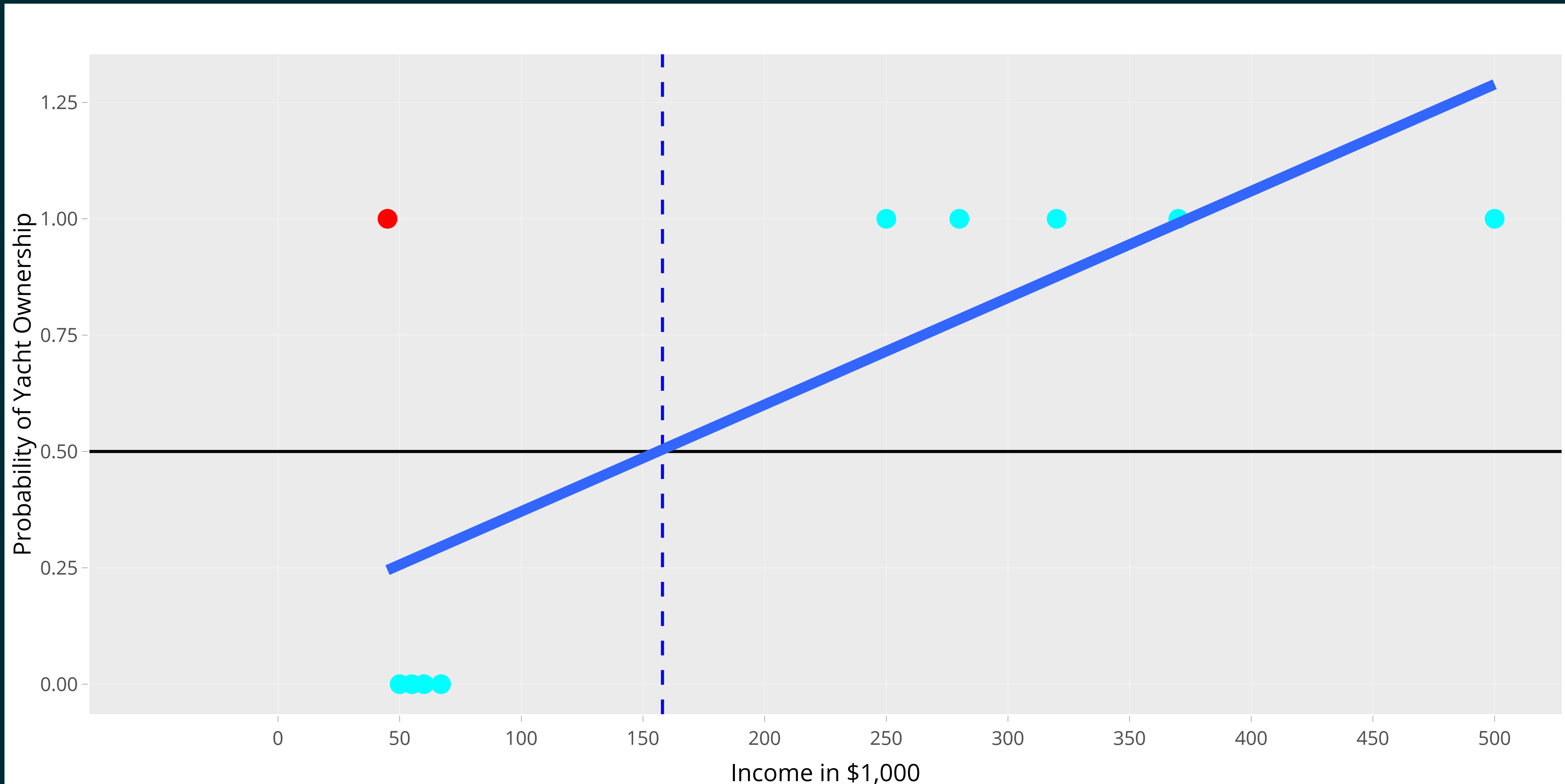


QUICK WAY TO FIND A DECISION BOUNDARY



1. Find the intersection point between the prediction line and the horizontal 0.5 probability line.
2. Draw a vertical line through the intersection point. This line is called a **decision boundary**.
3. All incomes left of the *decision boundary* (income smaller than 158) are predicted as “no”. All incomes right of the *decision boundary* (income greater than 158) are predicted as “yes”.»

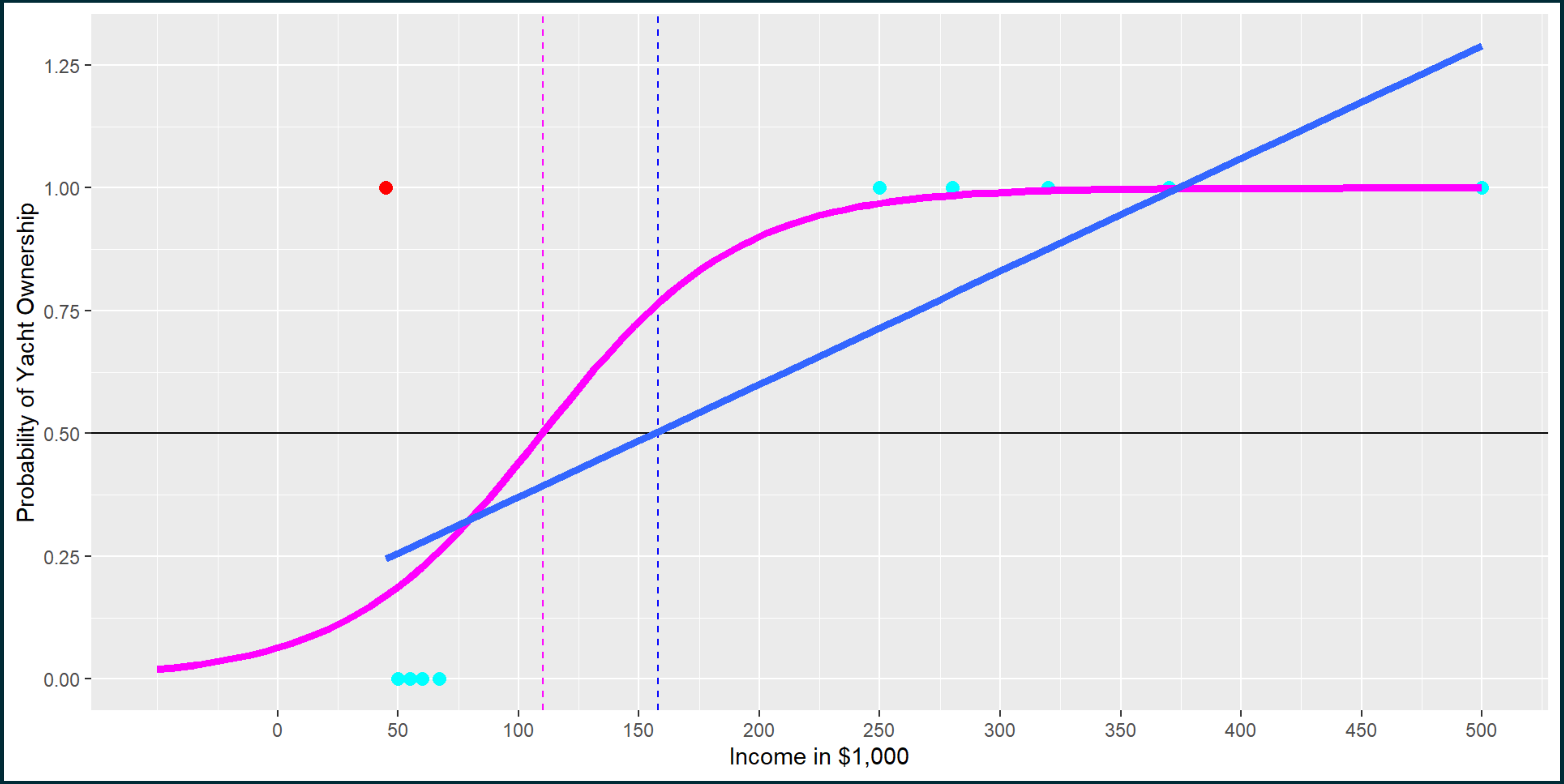
WHY OLS FOR CLASSIFICATION IS A BAD IDEA



Note, incomes > $370,000 are predicted with a probability > 100% to be yacht owners(?)
E.g. probability of owning a yacht for an income of $500,000 is 125% (?)

A similar problem can occur with negative probabilities!

A STEP-FUNCTION AS AN ALTERNATIVE TO OLS



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POPULAR STEP-FUNCTIONS (SIGMOID FUNCTION)

- The **Hyperbolic Tangent function**.
- The **Arc Tangent function**.
- The **Logistic function** (confusingly sometimes also called the *sigmoid function*).

$$y_i = \frac{1}{1 + e^{-x_i}}$$

»

THE LOGISTIC FUNCTION

- The **Logistic function** (confusingly sometimes also called the *sigmoid function*):

$$y_i = \frac{1}{1 + e^{-x_i}}$$

We use: $y_i = P^{rob}_{yes,i}$ and $x_i = \beta_1 Inc_i + \beta_2$ which gives us:

$$P^{rob}_{yes,i} = \frac{1}{1 + e^{-(\beta_1 Inc_i + \beta_2)}}$$

β_1 and β_2 change slope and position
$\beta_1 = 1$ and $\beta_2 = 0$ gives the org. logistic function. 🤓

WHAT MAKES THE LOGISTIC FUNCTION SO SPECIAL?

– COMPARED TO OTHER SIGMOID (STEP) FUNCTIONS –

Time for some mathematical magic:

Logistic function $P^{rob}_{yes,i}$:= probability for positive event (e.g. yacht ownership: yes):

$$P^{rob}_{yes,i} = \frac{1}{1 + e^{-(\beta_1 \cdot x_i + \beta_2)}}$$

Take the inverse on both sides of the equation:

$$\frac{1}{P^{rob}_{yes,i}} = 1 + e^{-(\beta_1 \cdot x_i + \beta_2)}$$

Subtract 1 on both sides:

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$$\frac{1}{P^{rob}_{yes,i}} - 1 = e^{-(\beta_1 \cdot x_i + \beta_2)}$$

Consider that $-1 = -\frac{P^{rob}_{yes,i}}{P^{rob}_{yes,i}}$ and substitute -1 accordingly, we get after simplification:

$$\frac{1 - P^{rob}_{yes,i}}{P^{rob}_{yes,i}} = e^{-(\beta_1 \cdot x_i + \beta_2)}$$

$1 - P^{rob}_{yes,i}$ equals by definition $P^{rob}_{no,i}$:

$$\frac{P^{rob}_{no,i}}{P^{rob}_{yes,i}} = e^{-(\beta_1 \cdot x_i + \beta_2)}$$

Take again the inverse on both sides:

$$\frac{P^{rob}_{yes,i}}{P^{rob}_{no,i}} = e^{\beta_1 \cdot x_i + \beta_2}$$

Take the logarithm on both sides:

$$\ln\left(\frac{P^{rob}_{yes,i}}{P^{rob}_{no,i}}\right) = \beta_1 \cdot x_i + \beta_2$$

ONE MORE STEP – ODDS VS PROBABILTIES

- The fraction of the yes/no probabilities can be interpreted as $Odds$ as they are often used in betting.
- Example: The probability of getting two heads when flipping two coins is is $P^{rob}_{yes,i} = 0.25$.
- Consequently, the probability of **not** getting two heads when flipping two coins is $P^{rob}_{no,i} = 0.75$.
- $Odds$ for 2 Heads compared to **not** 2 heads is 1 to 3 or 33%:

$$O^{dds} = \frac{P^{rob}_{yes,i}}{P^{rob}_{no,i}} = \frac{0.25}{0.75} = \frac{1}{3} = 0.33$$

INTERPRETATION OF THE βs: YACHT OWNERSHIP

$$\ln(O^{dds}) = \ln\left(\frac{P^{rob}_{yes,i}}{P^{rob}_{no,i}}\right) = 0.02 \cdot Inc_i + (-2.7)$$

Model results after running and printing the workflow():

▶ Code

```
══ Workflow [trained] ══════════════════════════════════════════════════════════
Preprocessor: Recipe
Model: logistic_reg()

── Preprocessor ────────────────────────────────────────────────────────────────
0 Recipe Steps

── Model ───────────────────────────────────────────────────────────────────────

Call:  stats::glm(formula = ..y ~ ., family = stats::binomial, data = data)

Coefficients:
(Intercept)       Income
   -2.68660      0.02448
```

```
Degrees of Freedom: 9 Total (i.e. Null);  8 Residual
Null Deviance:      13.46
Residual Deviance: 5.654     AIC: 9.654
```

INTERPRETATION OF THE βs: YACHT OWNERSHIP

$$\ln(O^{dds}) = \ln\left(\frac{P^{rob}_{yes,i}}{P^{rob}_{no,i}}\right) = 0.02 \cdot Inc_i + (-2.7)$$

- If income increases by 1 ($1,000) the logarithm of the odds increases by 0.02.
- Since change of a logarithm is a relative change (**percentage**):

If income increases by 1 ($1,000) the odds increases by 2% (0.02). (careful with the results because data were made up and N is too small!)

CONFUSION MATRIX

Note, in the mockup we did not create training and testing data. Therefore, we use DataYachts (the data we used to fit/train the workflow) here. This is not a proper methodology but good enough for the mock-up:

▶ Code

```
          Truth
Prediction 0 1
         0 4 1
         1 0 5
```

REAL WORLD CHURN ANALYSIS WITH LOGISTIC REGRESSION – THE DATA

We use data (7,043 customers) of the fictional telecommunication company *TELCO*, generated by *IBM* for training purposes:

- The outcome variable $Churn$ indicates, if a customer departed within the last month ($Churn = Yes$) or not ($Churn = No$).
- Predictor variables contain:
  - Customers' $Gender$ ($Female$ or $Male$),
  - Customers' $SeniorCitizen$ status (0 for no or 1 for yes),
  - Customers' $Tenure$ with *TELCO* (month of membership), as well as
  - Customers' $MonthlyCharges$ (in US-$).

REAL WORLD CHURN ANALYSIS WITH LOGISTIC REGRESSION – THE DATA

▶ Code

```
  Churn Gender SeniorCitizen Tenure MonthlyCharges
1    No Female             0      1          29.85
2    No   Male             0     34          56.95
3   Yes   Male             0      2          53.85
4    No   Male             0     45          42.30
5   Yes Female             0      2          70.70
6   Yes Female             0      8          99.65
```

REAL WORLD CHURN ANALYSIS WITH LOGISTIC REGRESSION

– DO IT YOURSELF –

Create the Churn analysis with logistic regression. Click on the link in the footer to get an R-script with a skeleton for the analysis. 🤓

RESULTS FROM CHURN ANALYSIS WITH LOGISTIC REGRESSION

Confusion Matrix:

	Yes	No
Yes	239	150
No	322	1403

Accuracy:

.metric	.estimator	.estimate
accuracy	binary	0.7767266

Sensitivity:

.metric	.estimator	.estimate
sensitivity	binary	0.426025

Specificity:

.metric	.estimator	.estimate
specificity	binary	0.9034127

Hint: What do the column sums of the confusion matrix tell you?

PROBLEM: UNBALLANCED TRAINING DATA

Churn	n
Yes	1308
No	3621

Majority Class: $Churn = No$ has 3621 observations in the training dataset.

Minority class $Churn = Yes$ has 1308 observations in the training dataset.

WHAT CAN WE DO?

Churn	n
Yes	1308
No	3621

PERFORMING DOWN-SAMPLING WITH `step_downsample()`

You need to add the R package `themis`. Then in your script, you can add `step_downsample(Churn)` to the recipe (don't forget to execute the following command lines again). As a reminder our original DataTrain had 4,929 observations, $Churn_{Yes} = 1308$, $Churn_{No} = 3621$:

▶ Code

Churn	n
Yes	1308
No	1308

PERFORMING UP-SAMPLING WITH `step_upsample()`

You need to add the R package `themis`. Then in your script, you can add `step_upsample(Churn)` to the recipe (don't forget to execute the following command lines again). As a reminder our original DataTrain had 4,929 observations, $Churn_{Yes} = 1308$, $Churn_{No} = 3621$:

▶ Code

Churn	n
Yes	3621
No	3621

Note, the number of observations has increased by 2313. The information in the dataset has not increased!

PERFORMING UP-SAMPLING WITH `step_smote()`. WHAT IS THE ADVANTAGE

As a reminder our original DataTrain had 4,929 observations, $Churn_{Yes} = 1308, Churn_{No} = 3621$:

▶ Code

Churn	n
Yes	3621
No	3621

Instead of copying a record from the training dataset, `step_smote()` finds the Nearest Neighbor to that record and creates a new record that has features generated as a weighted average between the Nearest Neighbor and the original record.